KUWAIT FOUNDATION LECTURE 13 - February 12, 2001

DIOPHANTINE EVOLUTION

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The intention of the talk was to discuss some work which we consider to be the millennium results in the past millennium in number theory with some emphasis on diophantine questions. They all date back to the time between 1900 and 2000. In the first part of this century one of the most important results was Siegel's theorem on integral points on curves and the second result is the finite basis theorem of Weil which had been obtained earlier in a special case by Mordell. Both, Siegel's and Weil's result were fundamental and conceptual breakthroughs in the theory.

In the second half of the century the most important contributions were made by Roth, Baker, Deligne, Faltings and Wiles. We discussed in detail their work and how the work is related. Two basic theories turned out to be absolutely crucial for getting finiteness results for rational and integral points on varieties. One is the theory of diophantine approximations which is already essential in the work of Siegel. The theory has been fundamentally developed in different ways by Roth, Baker, Faltings, Voijta, Bombieri, Masser and Wüstholz. The other theory is the theory of Galois representations, central in the work of Serre, Deligne, Faltings and Wiles. Both theories are different in an essential way. However there is a strong connection in form of the *abc*-conjecture. Very surprisingly this conjecture may be formulated in terms of logarithmic forms and is equivalent to adelic conjectural bounds for logarithmic forms. In its simplest version it states that if $\Lambda = u_1 \log v_1 + \cdots + u_n \log v_n$, $u_1, \ldots, u_n \in \mathbb{Z}$ not all 0, $v_1, \ldots, v_n > 0$ integral and $u = \max |u_j|$, then

$$\log \min(1, |\Lambda|) + \sum_p \log \min(1, |\Lambda|_p \cdot p) \gg -\log n(\log v_1 + \cdots + \log v_n)$$

(Baker's conjecture, weak form). Practically all the work we discussed follows from such a lower bound.

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