

Higher Dimensional Categories

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According to Eilenberg and Mac Lane, founders of Category Theory, categories were needed to explain functors and functors were needed to explain natural transformations. Thus from the beginning it was clear that the study of 1-dimensional categories required a 2-dimensional structure, a 2-category; and so on. In the simplest versions of the resulting higher dimensional categories, one takes composition to be strictly associative. I shall outline the theory and explain some results.

Unfortunately, for compelling applications the associativity of composition fails, and one needs a theory which treats ever more subtle notions of equivalence. One approach to this borrows ideas from homotopy theory. I shall try both to explain some concrete motivations and to convey the flavour of the subject as it appears to be developing.