Riemannian Geometry (L24)

Gabriel P. Paternain

This course is one of the possible natural continuations of the course *Differential Geometry* offered in the Michaelmas Term. Once Riemannian metrics and their various curvatures (i.e. sectional, Ricci and scalar) have been defined, the following general question stands out:

Given a certain property of the curvature tensor, which manifolds admit Riemannian metrics whose curvature has the given property?

A very concrete example could be: which complete manifolds admit Riemannian metrics with non-negative Ricci curvature?

This particular question is still wide open (as well as many others!), but there's a lot we know about it. In this course we will explore various techniques and results that will reveal an intricate network of subtle relations between curvature and topology.

Contents:

1. Brief review from the Differential Geometry course. Jacobi fields, completeness and the Hopf-Rinow theorem.
2. Classification of simply connected space forms. The Hadamard-Cartan theorem.
3. Variations of energy, Bonnet-Myers diameter theorem and Synge's theorem.
4. Ricci curvature comparison. Riemannian volume density. Cut locus. Bishop-Gromov volume comparison.
5. Maximal diameter theorem. Fundamental groups and Ricci curvature. Gromov's proof of Gallot's bound on the first Betti number.
6. The Cheeger-Gromoll splitting theorem.

Desirable Previous Knowledge

Manifolds, differential forms, flows and vector fields. Basic knowledge on local Riemannian geometry (curvature, geodesics etc.) and Lie groups. The course *Differential Geometry* offered in the Michaelmas Term is the ideal pre-requisite.

Introductory Reading

1. M.P. do Carmo, *Riemannan Geometry,* Boston: Birkhäuser, 1993.
2. S. Gallot, D. Hulin and J. Lafontaine, *Riemannian Geometry,* Berlin-Heidelberg: Springer-Verlag, 1987.

The first few chapters of reference 1 provide a good introductory reading.

Reading to complement course material

1. P. Petersen, *Riemannian Geometry,* Graduate Texts in Mathematics, Springer-Verlag, New York, 1998.

2. M.P. do Carmo, *Riemannan Geometry,* Boston: Birkhäuser, 1993.

3. S. Gallot, D. Hulin and J. Lafontaine, *Riemannian Geometry,* Berlin-Heidelberg: Springer-Verlag, 1987.

4. I. Chavel, *Riemannian Geometry, A modern Introduction,* CUP 1995.

There will be 3 example sheets and 3 example classes given by the lecturer.